DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

M.A/M.Sc. 2nd Semester

Name of Programme : M.A/M. Sc. Mathematics
Paper Type : Theory
Paper Code : MAT-510
Paper Title : Differential Equations-II
Full Marks : 80
Pass Marks : 32
Duration: 3 Hours

The figures in the margin indicate full marks for the questions

Answer all the questions:

Answer any three of the following questions: $10 \times 3 = 30$

1. a) Find the PDE of all spheres of radius λ ,having center in the $xy-$plane.
   b) Form PDE by eliminating ϕ from the equation $\phi(u,v)=0$.
   c) Solve the Cauchy Problem $p+q+2z=0$, $x_0=\mu$, $y_0=\mu$, $z_0=\sin\mu$.
2. Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.
3. a) Define Complete Integral, General Integral, Singular Integral of a given nonlinear PDE.
   b) Show that the equations $xp=yq$, $z(xp+yq)=2xy$ are compatible and solve them.
4. Show that the equation $xpq+yq^2=1$ has complete integrals $(a)\,(z+b)^2=4(ax+y)$ and $(b)\,kx(z+h)=k^2y+x^2$. And hence deduce (b) from (a).
5. Find a complete integral of $p_1^3+p_2^3+p_3^3=1$.

Answer any three of the following questions: **$10 \times 3 = 30$**

6. a) If $(\alpha D + \beta D' + \gamma)$ is a factor of $F(D, D')$ and $\phi(\xi)$ is an arbitrary function of the single variable ξ then, if $\alpha \neq 0$, prove that $z = e^{-\frac{\gamma}{\alpha}x}\phi(\beta x - \alpha y)$ is a solution of the equation $F(D, D')z = 0$.

   b) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

7. a) Show that the equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ possesses solutions of the form $\sum_{n=0}^{\infty} c_n \cos(nx + \varepsilon_n)e^{kn^2 t}$.

   b) Find a Particular Integral of
      i) $(D^2 - D')z = 2y - x^2$
      ii) $(D^2 - D')z = A\cos(lx + my)$.

8. Solve $r + 4s + t + (rt - s^2) = 2$
9. Solve $q^2 r - 2pqs + p^2 t = 0$

Answer any two of the following questions: **$10 \times 2 = 20$**

10. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.
11. Use Separation of Variables method to solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$; $u(0,t) = u(a,t) = 0$; $u(x,0) = f(x)$, $0 < x < a$
12. Discuss D' Alembert's solution of 1-dimensional wave equation.
13 Find the bounded solution of $\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = xt$ when $y = 0 = \frac{\partial y}{\partial t}$ at $t = 0$ and $y(0,t) = 0$.

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